## Lab 11: Rotational Dynamics

**Objectives**:
- To understand the relationship between net torque and angular acceleration.
- To understand the concept of the moment of inertia.
- To understand the concept of angular momentum.
- To understand that the angular momentum is conserved if there is zero net torque.
- To understand the relationship between net torque and angular momentum.

**Equipment**:
- 1 rotary motion sensor
- 1 rotational accessory
- string
- 1 small set of hanging masses
- 1 rod
- 1 rod stand
- 1 ring and disk set
- LoggerPro software
- LabPro computer interface
- Excel Program
- 1 rotary motion sensor

### Exploration 1 Motion and net torque

**Exploration 1.1** How could you keep an object rotating at constant angular speed? Would you apply a force? If so, in what direction would the force be applied? Would the force be continually applied? Explain your reasoning.

**Exploration 1.2** How could you keep an object rotating at constant angular acceleration? Would you apply a force? If so, in what direction would the force be applied? Would the force be continually applied? Explain your reasoning.

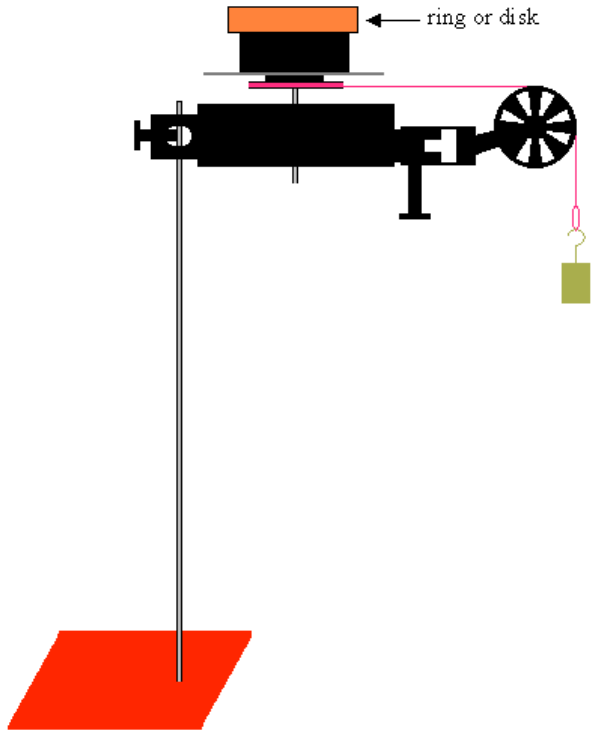
**Exploration 1.3** Would there be a net torque on the object in each of the cases Exploration 1.1.a and Exploration 1.1.b above?

**Exploration 1.4** Could you start an object rotating with zero net torque on the object? Explain.

**Exploration 2 Angular acceleration and net torque**

**Exploration 2.1** Consider two objects of equal mass and different shape. For example, consider a ring and a disk of equal mass. If the same net torque is applied to each object, would they have the same angular acceleration? (This is a prediction.)

**Exploration 2.2** Set up the following experiment to test your prediction in part Exploration 2.1. It consists of an object mounted on a platform that is free to rotate. A torque can be applied to the platform by a string attached to the platform (wrapped around the base of the platform), that runs over a pulley and is connected to a mass. If the mass is released, and allowed to fall, a torque is applied to the platform.




If there is very little friction on the platform, determine mathematically, in symbols, the net torque on the rotating platform containing the disk or ring. What measurements would you have to make to calculate the net torque? Show your work.

**Exploration 2.3** Does the platform accelerate at the same rate with the same net torque applied, if the ring is on top as if the disk is on top? Try it out.

**Exploration 2.4** How are angular acceleration and net torque related? (This is a prediction.)

**Investigation 1 Net torque and angular acceleration**

**Investigation 1.1** Using the same apparatus as in Exploration 2, perform the following experiment with the ring:

Connect the LabPro to the rotary motion sensor.

Using LoggerPro, record the number of revolutions of the ring plus platform in 0.5 seconds for 10g, 20g, 30g, 40g and 50g hanging masses.

| hanging mass | number of revolutions in 5.0s |
|---|---|
| 10g | |
| 20g | |
| 30g | |
| 40g | |
| 50g | |

**Investigation 1.2** Assuming very little friction on the platform, determine the net torque on the rotating ring plus platform for each mass. Show your work below.

| hanging mass | net torque |
|---|---|
| 10g | |
| 20g | |
| 30g | |
| 40g | |
| 50g | |

**Investigation 1.3** Using rotational kinematics equations, determine the angular acceleration for each net torque. Show your equations and calculations below.

| hanging mass | angular acceleration |
|---|---|
| 10g | |
| 20g | |
| 30g | |
| 40g | |
| 50g | |

**Investigation 1.4** Using Excel plot net torque (y-axis) vs. angular acceleration (x-axis).

Is there a relationship between the net torque and the angular acceleration? Compare to your prediction in Exploration 2.4.

**Investigation 2.** Repeat Investigation 1 with the disk.

**Investigation 2.1** Using LoggerPro, record the number of revolutions of the ring plus platform in 0.5 seconds for 10g, 20g, 30g, 40g and 50g hanging masses.

| hanging mass | number of revolutions in 5.0s |
|---|---|
| 10g | |
| 20g | |
| 30g | |
| 40g | |
| 50g | |

**Investigation 2.2** Assuming very little friction on the platform, determine the net torque on the rotating ring plus platform for each mass. Show your work below.

| hanging mass | net torque |
|---|---|
| 10g | |
| 20g | |
| 30g | |
| 40g | |
| 50g | |

**Investigation 2.3** Using rotational kinematics equations, determine the angular acceleration for each net torque. Show your equations and calculations below.

| hanging mass | angular acceleration |
|---|---|
| 10g | |
| 20g | |
| 30g | |
| 40g | |
| 50g | |

**Investigation 2.4** Using Excel plot net torque (y-axis) vs. angular acceleration (x-axis).

Is there a relationship between the net torque and the angular acceleration? Compare to your prediction in Exploration 2.4.

**Investigation 3 Moment of Inertia**

**Investigation 3.1** With both the disk and the ring, you probably found that the data fell in a straight line. For each case, in Excel determine the slope of the line: under the *Chart* option choose *Add Trendline*. Under *Type* choose *Linear*. Under *Options* choose *Display equation on chart* and *Display R-squared value on chart*. Determine the slope of the line for the ring and the disk. Are the slopes the same?

Slope for ring ___________________ Slope for disk __________________

**Investigation 3.2** Estimate the uncertainty in your slope measurements. How can you reasonably estimate the uncertainty based on your measurements.

Uncertainty in slope for ring __________________________

Uncertainty in slope for disk ___________________________

Discuss your method with your TA.

The net torque and the angular acceleration are related by a constant

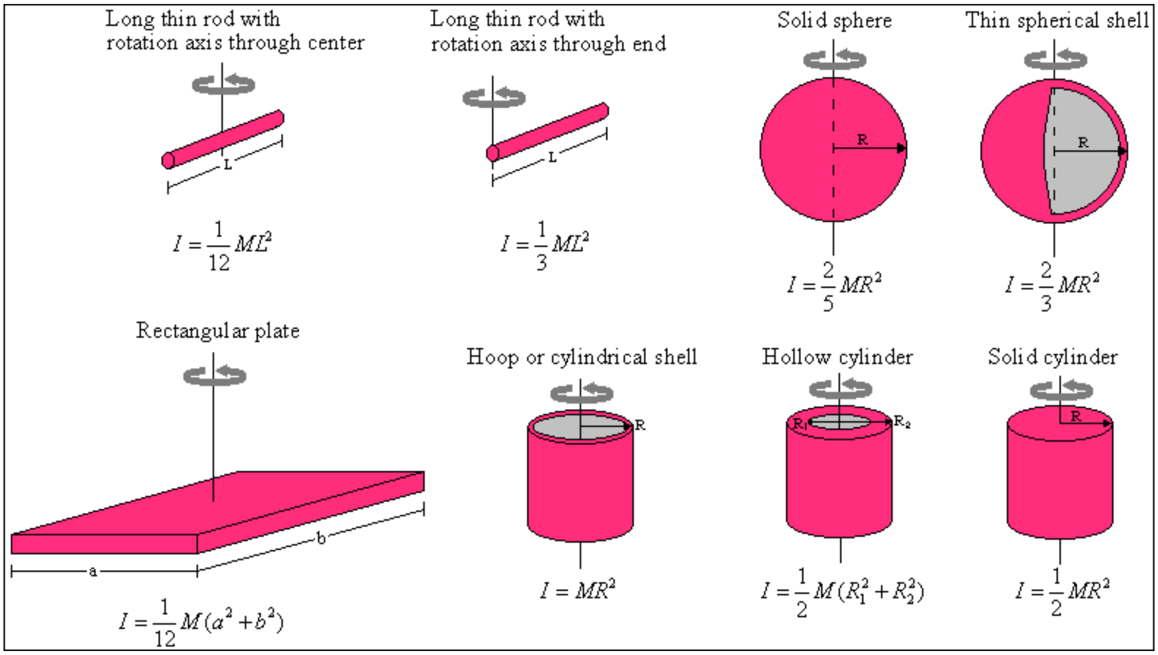
$$\tau_{net} = I\alpha$$

The constant I is called the moment of inertia. The units of the moment inertia are $kgm^2$.

The moment of inertia is different for different objects; it depends on how mass of the object is distributed and the axis about which the object is rotating. It can be found experimentally, as you did, by measuring the net torque and angular acceleration of an object about a particular axis. It is also possible to calculate the moment of inertia about a particular axis. The moment of inertia is given mathematically by

$$I = \sum_i m_i r_i^2$$

where the $m_i$ are the small pieces of mass in which the object is composed and the $r_i$ are the distances of the small pieces of mass to the axis of rotation. The moment of inertia for some common objects are given below.

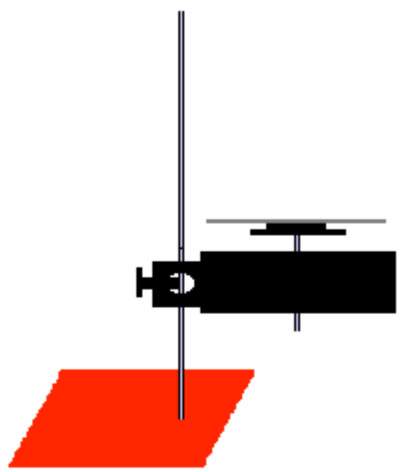



**Investigation 3.3** Compare your measured values of the moment of inertia, I, with the calculated values for the moment of inertia for the disk and the ring. Include your estimates of the uncertainty. Do they agree within the uncertainty?

**Exploration 4 Conservation of Angular momentum**

**Exploration 4.1** An object rotates with zero angular acceleration. Is there a net torque on the object? Explain.

**Exploration 4.2** Consider the following experiment. A platform, as in the picture below, is rotating at constant speed. An object, such as a ring or a disk is dropped onto the platform. Does the speed of rotation of the platform change? Does the net torque change? Does anything other measureable quantity change? Explain your reasoning for your predictions.

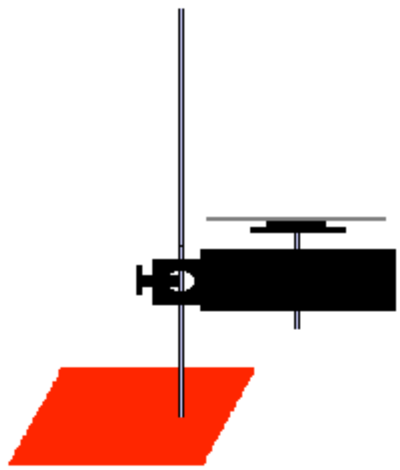


**Investigation 4 Conservation of Angular Momentum**

**Investigation 4.1.** Perform the experiment described in Exploration 4. Use the rotary motion sensor and the platform as shown in the picture in Exploration 4.

Start the platform spinning and then start taking revolutions vs. time data.

After the platform has been spinning for 2 - 3 seconds, still taking data, drop the black ring on top of the spinning platform (get as close as you can to the platform before you drop it).



**Investigation 4.2** Determine the angular velocity before and after the ring was dropped onto the platform from the LoggerPro graph.

Angular velocity before ring dropped ________________________

Angular velocity after ring dropped __________________________

**Investigation 4.3** Determine the moments of inertia for the platform and the ring. Show your calculations below.

Moment of inertia of ring ______________

Moment of inertia of platform __________________

Moment of inertia of ring + platform __________________

**Investigation 4.4.a** Calculate the product of the total moment of inertia before the loop was dropped, $I_1$, and the angular velocity before for the loop was dropped, $\omega_1$.

$I_1\omega_1$ ___________________________

Calculate the product of the total moment of inertia after the loop was dropped, $I_2$, and the angular velocity after for the loop was dropped, $\omega_2$.

$I_2\,\omega_2$ ___________________________

Compare the product of the total moment of inertia before the loop was dropped, $I_1$, and the angular velocity before for the loop was dropped, $\omega_1$, to the product of the total moment of inertia after the loop was dropped, $I_2$, and the angular velocity after for the loop was dropped, $\omega_2$.

Is the data consistent with your predictions in exploration 4?

If the net torque of an object is zero, the product of the total moment of inertia and the angular velocity, Iω, is constant:

$$I_1\omega_1 = I_2\omega_2$$

where $I_1$ is the initial moment of inertia, $I_2$ is the final moment of inertia, $\omega_1$ is the initial angular velocity and $\omega_2$ is the final angular velocity.

The product of the total moment of inertia and the angular velocity, Iω, is equal to the angular momentum, L ( $L = I\omega$ for a rotating object).

The torque is equal to the rate of change of angular momentum:

$$\boldsymbol{\tau} = d\mathbf{L}/d\tau$$

If there is no net torque on an object, the angular momentum is conserved; the initial angular momentum is equal to the final angular momentum. This can be written as:

$$\boldsymbol{L}_{final}^{net} = \boldsymbol{L}_{initial}^{net}$$

For a rotating object,

$$I_i\omega_i = I_f\omega_f$$